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THE TURBULENT FLOW OF A CONDUCTING FLUID IN A LONGITUDINAL MAGNETIC FIELD

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THE TURBULENT FLOW OF A CONDUCTING FLUID IN A LONGITUDINAL MAGNETIC FIELD

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ABSTRACT: A hypothesis with respect to a term describing the Joule dissipation of turbulent energy and taking the change in the structure of the turbulence of a flow in a magnetic field into consideration is formulated.

The use of equations for the Reynolds stress tensor (turbulent /10* energy balance equations), closed by semiempirical relationships have proven fruitful in investigating flows with a transverse shear in conventional hydrodynamics [1, 2, 7].

In magnetic hydrodynamic, the equations for the Reynolds stress tensor are supplemented by terms that take into consideration the influence of the magnetic field on the turbulence characteristics, and for which the semiempirical concept is a difficult one because of the limitations of the experimental material.

Nevertheless, attempts have been made [3, 4] to construct a semiempirical theory of turbulence based on the equations for the Reynolds stress tensor in the case of a flow of a conducting fluid in a longitudinal magnetic field where the influence of the field is taken into consideration by the terms describing the Joule dissipation of the turbulent energy, and for which the corresponding representations are used.

This paper follows [3, 4], and formulates a hypothesis with respect to a term describing the Joule dissipation of turbulent energy that has a simple physical concept while taking into consideration the change that takes place in the structure of the turbulence of a flow in a magnetic field. The use of this hypothesis, together with known Rotta relationships [2], for the rest of the terms in the equations for the Reynolds stress tensor, results in an expression for eddy viscosity that is analogous to the known Prandtl relationship and differs from the latter only in the coefficient that takes the

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* Numbers in the margin indicate pagination in the foreign text.

influence of the magnetic field into consideration.

1. In the Navier-Stokes equations[1]

$$\frac{\partial U_i}{\partial t}+U_k\frac{\partial U_i}{\partial x_k}=-\frac{1}{\rho}\frac{\partial P}{\partial x_i}+\nu\frac{\partial^2 U_i}{\partial x_k\,\partial x_k}+\frac{1}{\rho}X_i \quad (i=1,2,3) \tag{1.1}$$

the volume density of the external forces, X_i, in the case of motion of the electrically conducting fluid in a magnetic field has the known form

$$X_i=\varepsilon_{irs}J_rB_s \tag{1.2}$$

Here ε_{irs} is a completely antisymmetrical tensor of rank three, and J_r and B_s are components of the vectors of the induced electrical current, and of the magnetic field induction, respectively.

Representing the individual values of the fields for the corresponding magnitudes contained in the Navier-Stokes equations (1.1) in the form of a sum of mean and pulsation values, $U_i = \bar{U}_i + u_i$, for example, Eq. (1.1) and the equations of continuity for the mean and pulsation magnitudes can be written in the following form with Eq. (1.2) taken into consideration

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$$\frac{\partial \bar{U}_i}{\partial t}+\frac{\partial u_i}{\partial t}+\bar{U}_k\frac{\partial u_i}{\partial x_k}+u_k\frac{\partial \bar{U}_i}{\partial x_k}+\frac{\partial}{\partial x_k}(\bar{U}_i\bar{U}_k+u_iu_k)=$$
$$=-\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x_i}-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\nu\frac{\partial^2 \bar{U}_i}{\partial x_k\,\partial x_k}+\nu\frac{\partial^2 u_i}{\partial x_k\,\partial x_k}+\frac{1}{\rho}\varepsilon_{irs}(\bar{J}_r+j_r)(\bar{B}_s+b_s) \tag{1.3}$$

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1. Here, and in what follows, it is assumed that summing is from 1 to 3 with respect to the twice repeating index, for example,

$$U_k\frac{\partial U_i}{\partial x_k}=\sum_{k=1}^{3}U_k\frac{\partial U_i}{\partial x_k}.$$

Then, after averaging the terms in Eq. (1.3) and using the Reynolds condition [1], the result will be equations analogous to the Reynolds equations in conventional hydrodynamics

$$\frac{\partial \bar{U}_i}{\partial t} + \frac{\partial}{\partial x_k}(\bar{U}_i \bar{U}_k + \langle u_i u_k \rangle) = \\ = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x_i} + \nu \frac{\partial^2 \bar{U}_i}{\partial x_k \partial x_k} + \frac{1}{\rho}\varepsilon_{irs}(\bar{J}_r \bar{B}_s + \langle j_r b_s \rangle) \tag{1.4}$$

The procedure is known (see [1, 2], for example) for using Eqs. (1.3) and (1.4) to obtain the equations for the Reynolds stress tensor

$$\frac{\partial}{\partial t}\langle u_i u_j \rangle + \bar{U}_k \frac{\partial}{\partial x_k}\langle u_i u_j \rangle + \langle u_j u_k \rangle \frac{\partial \bar{U}_i}{\partial x_k} + \langle u_i u_k \rangle \frac{\partial \bar{U}_j}{\partial x_k} = \\ = \frac{1}{\rho}\left\langle p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right\rangle + \frac{\partial}{\partial x_k}\left[\nu \frac{\partial}{\partial x_k}\langle u_i u_j \rangle - \langle u_k u_i u_j \rangle - \right. \\ \left. - \frac{1}{\rho}\langle p(\delta_{jk} u_i + \delta_{ik} u_j)\rangle\right] - 2\nu\left\langle \frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}\right\rangle + \\ + \varepsilon_{irs}\frac{1}{\rho}[\bar{J}_r \langle b_s u_j \rangle + \bar{B}_s \langle u_j j_r \rangle + \langle j_r b_s u_j \rangle] + \\ + \varepsilon_{jrs}\frac{1}{\rho}[\bar{J}_r \langle b_s u_i \rangle + \bar{B}_s \langle u_i j_r \rangle + \langle j_r b_s u_i \rangle] \tag{1.5}$$

Here δ_{ij} is a unit tensor ($\delta_{ij} = 1$ when $i = j$, and $\delta_{ij} = 0$ when $i \neq j$).

If we assume that $i = j$ in Eq. (1.5), we obtain an equation for the turbulent energy balance $E = \langle u_i^2 \rangle / 2$, describing the change in the energy density for the pulsation motion with respect to time

$$\frac{\partial E}{\partial t} + \bar{U}_k \frac{\partial E}{\partial x_k} + \langle u_i u_k \rangle \frac{\partial \bar{U}_i}{\partial x_k} = \frac{\partial}{\partial x_k}\left[\nu \frac{\partial E}{\partial x_k} - \frac{1}{2}\langle u_i^2 u_k \rangle - \frac{1}{\rho}\delta_{ik}\langle p u_i \rangle\right] - \\ - \nu\left\langle \left(\frac{\partial u_i}{\partial x_k}\right)^2 \right\rangle + \frac{1}{\rho}\varepsilon_{irs}[\bar{J}_r \langle b_s u_i \rangle + \bar{B}_s \langle j_r u_i \rangle + \langle j_r b_s u_i \rangle] \tag{1.6}$$

All the terms in Eq. (1.6) have an explicit physical concept. Specifically, the last term is the Joule dissipation of the turbulent energy, the result of the pulsation motion of the fluid in the magnetic field[2].

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2. See [1, 2], for example, concerning the physical concept for the rest of the terms in Eq. (1.6).

It is obvious that the equations for the Reynolds stress tensor in the form of Eq. (1.5) cannot be solved with respect to $< u_i u_j >$ in the general form. However, there are cases when extensive simplification of Eq. (1.5) is permissible, and can be accomplished.

2. Let us give similar [3, 4] consideration to a steady flow of a conducting fluid in a longitudinal magnetic field when there is equality between the generation and dissipation of turbulent energy, and with the redistribution of the energy along the coordinate axes taken into consideration; that is, we will ignore the convection and diffusion terms in Eq. (1.5). As the experiment in the case of absence of a magnetic field indicates, this occurs in most of the flow in a pipe [5], and in the boundary layer on a flat plate in the absence of a pressure gradient in the external flow [6]. Further, it is assumed that the magnetic Reynolds number, fixed with respect to the pulsation component of the flow velocity, is small enough that the pulsation component of the magnetic field induction can be ignored as compared with the mean magnitude of the induction for the field created by external sources. /12

So far as the terms describing the exchange of energy between the pulsations along the various coordinate axes, and the viscous dissipation of the energy of the pulsation motion in Eq. (1.5), are concerned, let us take Rotta's assumptions[3]

$$\frac{1}{\rho}\left\langle p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right\rangle = -k\frac{\sqrt{E}}{l}\left(\langle u_i u_j\rangle - \frac{2}{3}\delta_{ij}E\right) \tag{2.1}$$

$$2\nu\left\langle \frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}\right\rangle = \nu c_1\frac{\langle u_i u_j\rangle}{l^2}+\frac{2}{3}c\delta_{ij}\frac{E^{3/2}}{l} \tag{2.2}$$

Here, k, c, and c_1 are empirical constants, l is the scale of the pulsation motion for which the Prandtl mixing length is used in the absence of a magnetic field.

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3. It should be pointed out that in Eq. (2.1), moreover, the dependence of the pulsations in pressure on the magnetic field, is not taken into consideration in the case reviewed here because the pulsations are approximate, even in conventional hydrodynamics [1].

The following assumption is made with respect to the pulsation component of the electrical current resulting from the turbulent motion of the conducting fluid in a magnetic field.

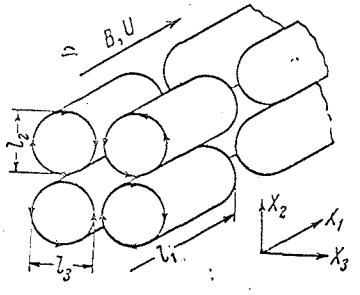

Figure 1.

Let the structure of the turbulent flow be characterized by vortices with various scales (mixing lengths) along the coordinate axes, x_1, x_2, and x_3 (Figure 1), but at the same time equality of the ratios of the scales to the corresponding pulsation velocities

$$\frac{l_1}{\sqrt{\langle u_1^2\rangle}}=\frac{l_2}{\sqrt{\langle u_2^2\rangle}}=\frac{l_3}{\sqrt{\langle u_3^2\rangle}} \qquad (2.3)$$

following from an evaluation of the order of magnitudes in the equation of continuity for the pulsation velocities, is satisfied.

What is being considered is the flow in a longitudinal magnetic field (that is, $\bar{B}_2 = \bar{B}_3 = 0$, $\bar{B}_1 = B$), so the electromotive force (emf) of induction occurs as a result of the interaction with the field of the transverse (with respect to the magnetic field) components of the pulsation velocity vector, u_2 and u_3, which as in the case of absence of a magnetic field [5, 6], we will take as approximately equal to each other [that is, $u_2 \approx u_3$, and as a result, $l_2 \approx l_3$, in accordance with Eq. (2.3)]. The pulsation electrical current thus appearing closes in a longitudinal (along the magnetic field) direction in such a way that the summed length of the current loop L_k is approximately equal in magnitude to the length of the vortex loop $2(l_1 + l_2)$.

The total emf, $\Delta\varphi$, generated in a current loop of length L_k can be represented in the form

$$\Delta\varphi = 2\alpha l_2 u_2 B \qquad (2.4)$$

where

α is a coefficient that takes into consideration the reduction in the emf as a result of the induced pulsation electrical field ($0 < \alpha < 1$).

The electrical current caused to flow by this emf in the direction along the magnetic field flows through a layer of fluid with section $l_2^2/4$, such that the total resistance of the loop is equal to

$$\rho_k = L_k / (\sigma l_2^2 / 4) = 8(l_1 + l_2) / \sigma l_2^2,$$

and the total current flowing in the loop is

$$J_k = \Delta\varphi / \rho_k = \alpha B u_2 l_2^3 \sigma / 4(l_1 + l_2)$$

Here σ is the conductivity of the fluid. Accordingly, when Eq. (2.3) is taken into consideration, the current density in the longitudinal direction is in the following form

$$j_1 = \frac{J_k}{l_2^2/4} = \alpha \frac{\sigma B u_2}{l_1/l_2 + 1} = \alpha \frac{\sigma B u_2}{(\langle u_1^2\rangle/\langle u_2^2\rangle)^{1/2} + 1} \tag{2.5}$$

The total current flowing in the loop must be divided by the area of the section through which this current is flowing in order to determine the current density in the transverse direction; that is,

$$j_2 \approx j_3 = \frac{J_k}{l_1 l_2/4} = \alpha \frac{\sigma B u_2}{(l_1/l_2)^2 + l_1/l_2} = \alpha \frac{\sigma B u_2}{\langle u_1^2\rangle/\langle u_2^2\rangle + (\langle u_1^2\rangle/\langle u_2^2\rangle)^{1/2}} \tag{2.6}$$

Thus, in the case considered, Eqs. (2.1), (2.2), (2.5), and (2.6) close the equations for the Reynolds stress tensor at (1.5), which in generally accepted notations

$$\overline{U}_1 = U, \quad u_1 = u, \quad u_2 = v, \quad x_1 = x, \quad x_2 = y$$

take the form

$$\langle uv\rangle \frac{dU}{dy} + \frac{k}{2}\frac{\sqrt{E}}{l}\left(\langle u^2\rangle - \frac{2}{3}E\right) + \nu \frac{c_1}{2}\frac{\langle u^2\rangle}{l^2} + \frac{c}{3}\frac{E^{3/2}}{l} = 0 \tag{2.7}$$

$$\frac{k}{2}\frac{\sqrt{E}}{l}\left(\langle v^2\rangle - \frac{2}{3}E\right) + \nu \frac{c_1}{2}\frac{\langle v^2\rangle}{l^2} + \frac{c}{3}\frac{E^{3/2}}{l} + \alpha \frac{\sigma B^2}{\rho}\frac{\langle v^2\rangle}{\langle u^2\rangle/\langle v^2\rangle + (\langle u^2\rangle/\langle v^2\rangle)^{1/2}} = 0 \tag{2.8}$$

$$\langle v^2\rangle \frac{dU}{dy} + k\frac{\sqrt{E}}{l}\langle uv\rangle + \nu c_1 \frac{\langle uv\rangle}{l^2} + \alpha \frac{\sigma B^2}{\rho} \frac{\langle uv\rangle}{\langle u^2\rangle/\langle v^2\rangle + (\langle u^2\rangle/\langle v^2\rangle)^{1/2}} = 0 \qquad (2.9)$$

In the case under consideration, the Reynolds Eq. (1.4) takes the known form

$$\frac{d}{dy}\langle uv\rangle = -\frac{1}{\rho}\frac{dP}{dx} + \nu\frac{d^2U}{dy^2} \qquad (2.10)$$

3. Eqs. (2.7) - (2.9) can be used to establish the connection between the Reynolds stress and the velocity gradient in the case of flow outside the viscous sublayer; that is, the term that takes into account the direct action of the viscosity on large-scale pulsations in the Rotta ratio for viscous dissipation at (2.2) can be ignored. /14

Thus, setting $c_1 = 0$, and in accordance with [3], introducing the anisotropy factor for the flow $\xi = \langle u^2\rangle / \langle v^2\rangle$, the following relationship can be obtained from Eqs. (2.7) - (2.9)

$$\langle uv\rangle = -\psi l^2 \left|\frac{dU}{dy}\right| \frac{dU}{dy} \qquad (3.1)$$

where

$$\psi = A_1^{3/2}\left[\sqrt{1 + (A_2 S_l)^2} - A_2 S_l\right]$$

$$A_1 = \frac{12}{(\xi+2)\left[\xi(k/c-1) + 5k/c - 2\right] c^{1/2}}, \qquad (3.2)$$

$$A_2 = \frac{\alpha}{\xi + \xi^{1/2}}\left(\frac{\xi(k/c-1) + 5k/c - 2}{3(\xi+2)}\right)^{1/2}$$

The coefficient ψ is a function of the local Stewart number

$$S_l = \frac{\sigma B^2}{\rho}\frac{1}{dU/dy}$$

and ξ depends implicitly on the local Steward number

$$S_l = \frac{\xi + \xi^{1/2}}{\alpha}\frac{\xi(k/c-1) - (k/c+2)}{\sqrt{3 + 2\left[\xi(k/c-1) - (k/c+2)\right]/(\xi+2)}} \qquad (3.3)$$

It is required that $\psi = 1$ when $S_l = 0$ in order that the expression for $\langle uv\rangle$ in Eq. (3.1) be converted into the known Prandtl formula when there is no magnetic field. This superposes a definite connection on the constants

k and c, specifically ($\xi = \xi_0$ when B = 0)

$$\frac{12}{(\xi_0+2)[\xi_0(k/c-1)+5k/c-2]\,c^{1/3}}=1 \tag{3.4}$$

On the other hand, from the dependency for $\xi = \xi(S_l)$ in Eq. (3.3) when $S_l = 0$, we can obtain a second relationship for the constants k and c

$$\xi_0(k/c-1)-(k/c+2)=0 \tag{3.5}$$

The anisotropy factor ξ_0 in Eqs. (3.4) and (3.5) is a new empirical constant [3] (replacing the k and c used earlier) with a distinct physical concept (the relationship of $\langle u^2\rangle$ to $\langle v^2\rangle$ when the effect of viscosity and of the magnetic field is lacking and conditions are such that there is equilibrium between the generation and dissipation of turbulent energy).

Therefore, Eqs. (3.4) and (3.5) will yield the following expressions for k and c

$$\frac{k}{c}=\frac{\xi_0+2}{\xi_0-1}, \qquad k=\left[\frac{8}{(\xi_0+2)^2(\xi_0-1)}\right]^{1/4} \tag{3.6}$$

and what follows is that for values ξ_0 = 1.5 to 3 (corresponding to the experimental data [5, 6] in the flow region under consideration) k/c = 7 to 2.5. The computations made in [7] with respect to the distribution of the pulsation characteristics of a pipe flow on the basis of Eqs. (2.7) - (2.9) (when B = 0) in the range of change in k/c = 10 to 1.6 revealed that the best coincidence with the experiment occurs when $k/c \approx 7$. /15

Once Eq. (3.6) for k and c is taken into consideration, Eqs. (3.2) and (3.3) for ψ and $\xi(S_l)$, respectively, can be transformed into the form

$$\psi=\left(\frac{2\vartheta}{1+\vartheta^{-1}}\right)^{3/2}\left[\sqrt{1+\left(\frac{\alpha}{\xi+\xi^{1/2}}\frac{1+\vartheta}{\xi_0-1}S_l\right)^2}-\frac{\alpha}{\xi+\xi^{1/2}}\frac{1+\vartheta}{\xi_0-1}S_l\right], \tag{3.7}$$

$$\vartheta=\frac{\xi_0+2}{\xi+2}$$

$$S_l=\frac{\xi+\xi^{1/2}}{\alpha}\frac{\xi/\xi_0-1}{\sqrt{(\xi/\xi_0+4/\xi_0+1)(\xi+\xi/\xi_0-2/\xi_0)}} \tag{3.8}$$

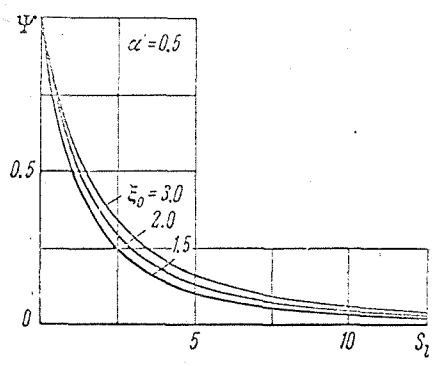

Figure 2.

Figure 2 shows $\psi = \psi(S_l)$ for a series of values for the empirical constant ξ_o. As will be seen, change in ξ_o in the limits from 1.5 to 3.0 has little effect on the form of the dependence of ψ on S_l, so a constant value for ξ_o equal to 2 can be used in the computations.

So far as the mixing length contained in the expression for the Reynolds stress, Eq. (3.1), is concerned, a hypothesis [3] is taken as the first approximation wherein the distribution of the transverse turbulence scale l_2 (corresponding to mixing length l) is considered as not dependent on the effect of the magnetic field, but instead as dependent on the geometry of the flow, that is, $l_2 = l$ is taken as if there is no magnetic field and assumes that the dimensions of the energy-carrying vortices (along the $x_2 = y$ axis) are always proportional to the width of the flow (to the distance between the walls in the channel, to the thickness of the boundary layer).

Thus, since we have a magnetohydrodynamic analog for the Prandtl hypotheses for Eq. (3.1) for Reynolds stress, the computation for the mean flow can be carried out.

4. Let us consider, by way of an example, the steady flow of a conducting fluid in a round pipe of radius r in a longitudinal magnetic field in the case of sufficiently large Reynolds numbers.

Integrating the Reynolds Eq. (2.10) from 0 to y/r for the condition $\tau/_{y=o} = \tau_w$, and designating $\sqrt{\tau_w/\rho} = u_*$, we obtain

$$-\langle uv\rangle + \nu\frac{dU}{dy} = u_*^2\left(1-\frac{y}{r}\right) \qquad (4.1)$$

Using Eq. (3.1) for $\langle uv \rangle$, and passing to the universal coordinates $\eta = yu_* / \nu$ and $\varphi = U / u_*$, we can rewrite Eq. (4.1) in the form

$$\psi l^2 R_*^2 \varphi'^2 + \varphi' = 1 - \eta / R_* \qquad (4.2)$$

Here the prime designates the derivatives with respect to η, and $R_* = ru_* / \nu$ is the Reynolds number, determined with respect to the dynamic velocity and the pipe radius.

The local Stewart number, the function of which is the factor ψ, will take the following form in universal variables

$$S_i = \left(\frac{H}{2R_*}\right)^2 \frac{1}{\varphi'}$$

The velocity profile $\varphi = \varphi(\eta)$ was computed on the assumption of a two-parameter flow model; that is, $\varphi' = 1 - \eta / R_*$ was taken[4] for $\eta \leq 11.7$, and the value of φ' was sought for $11.7 < \eta \leq R_*$ by the simultaneous solution of Eqs. (4.2), (3.7), and (3.8), using the successive approximations method. And the Prandtl-Nikuradze formula /16

$$\frac{l}{R_*} = 0.14 - 0.08\left(1 - \frac{\eta}{R_*}\right)^2 - 0.06\left(1 - \frac{\eta}{R_*}\right)^4 \qquad (4.3)$$

was used for the mixing length in Eq. (4.2).

The coefficient of hydraulic resistance in the case of a flow in a pipe is equal to $\lambda = S/\varphi_0^2$, where

$$\varphi_0 = \frac{2}{R_*}\int_0^{R_*}\left(1 - \frac{\eta}{R_*}\right)\varphi\, d\eta$$

and R_* is connected with the Reynolds number $R = 2rU_0/\nu$ by the relationship

$$R = 2\varphi_0 R_*$$

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4. It is obvious that the assumption with respect to the constancy of the thickness of the viscous sublayer does not take into consideration its growth in a longitudinal magnetic field (since $c_1 = 0$). This can lead to some increase in the resistance coefficient in the case of large Hartmann numbers, but will not change the results of the computation qualitatively.

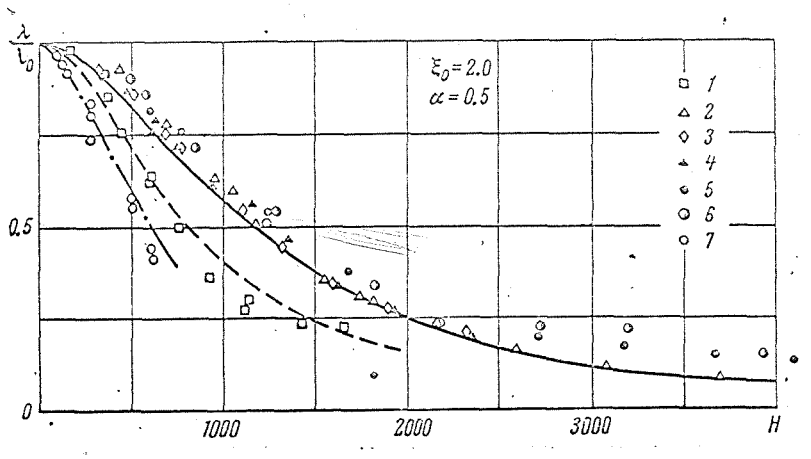

Figure 3.

The results of the computation for the resistance coefficient λ, equated to the resistance coefficient λ_o in the case of absence of a magnetic field, as a function of the Hartmann number H, are shown by the lines in Figure 3. The experimental data in [8, 9] are also plotted as points. Points 1, 2, 3, 4, 5, 6, and 7 correspond to the values for the Reynolds number $R \cdot 10^{-4} =$ 3.86, 7.41, 7.51, 7.85, 8.82, 9.15, (1.9 - 2.1); the dashed line is for $R = 3.86 \cdot 10^4$; the solid line is for $R = 7.85 \cdot 10^4$; and the dash-dot line is for $R = 2 \cdot 10^4$.

As will be seen from Figure 3, the coincidence of the results obtained from the computations and those from the experiment are entirely satisfactory in the region of Reynolds numbers shown. However, when $R < 2 \cdot 10^4$, where the flow does not have an adequately developed turbulent structure, generally speaking, comparison of the computation with the experiment revealed a marked divergence in results (the computation yields increased values for the resistance coefficient). Some divergence is also noted at very large Hartmann numbers, where the flow condition, as in the case of small Reynolds numbers, is close to laminar. This shows that consideration of the effect of the magnetic field on the flow of a conducting fluid in terms of just the mechanism involved in Joule dissipation of turbulent energy obviously is inadequate for purposes of obtaining the passage to the limit for the laminar flow condition, and that it is necessary as well to take into consideration the

effect of the field on the turbulence scale. However, lack of dependable results from the experimental investigation made of the structure of the turbulence in a magnetic field, as well as of an acceptable physical model of the effect of the field on the turbulence scale, does not permit doing so at this time.

In conclusion, the authors wish to express their thanks to V. M. Iyevlev, /17 I. G. Panevin, and S. A. Regirer for their discussion of the work and for their valuable advise, as well as to V. B. Levin for his kindness in allowing us to use the tables of results of the experiment.

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REFERENCES

1. Monin, A. S., and A. M. Yaglom, Statisticheskaya gidromekhanika. Mekhanika turbulentnosti [Statistical Hydromechanics. The Mechanics of Turbulence], Part I, Moscow, "Nauka" Press, 1965.

2. Rotta, J., "Statistical Theory of Nonhomogeneous Turbulence," Z. Physik, Vol. 129, No. 6, 1951, pp. 547-572.

3. Iyevlev, V. M., Teoreticheskoye issledovaniye vliyaniya magnitnogo polya na turbulentnost' potoka elektroprovodnoy zhidkosti [Theoretical Investigation of the Effect of a Magnetic Field on the Turbulence of the Flow of an Electrically Conducting Fluid]. Synposium on Problems of Turbulent Flows, Including Geophysical Applications. Abstracts of Reports and Communications, Kiev. "Naukova dumka" Press, 1967.

4. Kovner, D. S., and V. B. Levin, "The Turbulent Flow of an Electrically Conducting Fluid in a Pipe in a Longitudinal Magnetic Field," Tepolfizika vysokikh temperatur, Vol. 12, No. 5, 1964.

5. Laufer, J.,"The Structure of Turbulence in Fully Developed Pipe Flow," NACA Techn. Rep., No. 1174, 1954.

6. Klebanoff, P. S., "Characteristics of Turbulence in a Boundary Layer with Zero Pressure Gradient," NACA Techn. Notes, No. 3178, 1954.

7. Levin, V. B., "The Computation for the Fundamental Characteristics of Turbulent Flows with Transverse Shear," Tepolofizika vysokikh temperatur, Vol. 2, No. 4, 1964.

8. Genin, L. G., Zhilin, V. G., and B. S. Petukhov, "Experimental Investigation of a Turbulent Flow of Mercury in a Round Pipe in a Longitudinal Magnetic Field," Teplofizika vysokikh temperatur, Vol. 5, No. 2, 1967.

9. Levin, V. B., and I. A. Chinenkov, "Experimental Investigation of the Turbulent Flow of an Electrically Conducting Fluid in a Pipe in a Longitudinal Magnetic Field," Magnitnaya gidrodinamika, No. 4, 1966.

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